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Accuracy Improvement for Linear Tetrahedral Finite Element by Means of Virtual Mesh Refinement

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ABSTRACT

This paper offers a new strategy to improve accuracy on deformation field of linear 4-node tetrahedral finite elements (FE) commonly used in computer aided engineering (CAE) software by means of virtual mesh refinement (VMR). Here, the improvement is achieved by embodying additional virtual strain energy to the FE instead of using physical mesh refinement in particular domains. In this early development only total strain energy of elements in the domains after simulation is used to assess convergence during the VMR process. Finally, two benchmarking studies, i.e., classical cantilever beam and thin shell torsion, are used to test the ability of the method to solve shear dominated problems in large and small deformation, respectively. From the studies, improvements in the deformation accuracy by order of 200% are observed.

Keywords: *Tetrahedral, finite elements, VMR, strain energy, convergence*

Introduction

Nowadays, computer aided engineering (CAE) software relied on finite element method (FEM) has been considered as one of the best tool to achieve optimum design processes and parameters in particular for mechanical equipments or machine elements, as described in [1] for instance. Based on this fact, it has been major interest to FEM researchers to continuously increase its capability and robustness for the designers especially for them with less experience in FEM. Furthermore, as computer memory and speed will increase in the future, solid elements representing solid modeling approach will be used more frequently

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for their more realistic appearances than dimensionally reduced elements, e.g., beams, plates or axissymmetry elements.

To the best of author knowledge most of CAE software in the market offer auto meshing technique in FEM discretization for direct solid modeling. For complex geometries, it is often associated with an option to use tetrahedral finite element through domain in interest. Two tetrahedral elements commonly found herein are linear element with 4-nodes and quadratic element 10-nodes. The linear element is well known for its simpler formulation for programming implementation than the quadratic one however it has drawback to lock deformation shape to linear only. Hence, to successfully solve problems involving bending and stress concentration one needs very large number of these elements. On the other hand, by using the quadratic element both of problems can be easily solved as described by [2], for instance.

Since formulation of the linear tetrahedral element is often known to obtain faster computational time than the quadratic one, many techniques to improve accuracy of its deformation behaviour have been developed by several authors. A technique called mixed enhanced which is applicable for both small and large deformation in particular for incompressible continua has been developed in [3]. Relied on deformation based 4-node tetrahedral element, Payen and Bathe in [4] have shown that stress distribution on that element can be improved without modification in the deformation accuracy itself but using the nodal point force (NPF) based stresses method. Some recent articles regarding the improvement are relied on the popular smoothed method proposed by [5], non-conforming formulation with a special shape function on finite element space in [6], interpolation cover scheme [7] and the Petrov-Galerkin method [8]. Unfortunately, the aforementioned formulations and methods as well as other similar approaches are mathematically difficult to most of CAE engineers because they involve abstract mathematical analysis and definition. Thus, requirement on more intuitive approach to improve the accuracy of the element is of particular interest.

This work offers a novel approach to fulfill such demand namely virtual mesh refinement (VMR) method. Instead of relying on rigorous mathematical expression as the predecessors, it basically uses numerical experiments to capture behaviour of the tetrahedral element under specified boundary condition. The experiment is inspired by common physical mesh refinement on a domain which yields strain energy increment for smaller mesh sizes. If tetrahedron volume is considered as representation of such domain, its strain energy increases depending on number of element used and independent to applied boundary conditions. Here, the increment is then virtually incorporated to original formulation of the linear element to properly modify its particular deformation behavior. For the experiment, only the Total Lagrange formulation with additional pressure degree of freedom or mixed u/p formulation is used either for small or large deformation.

The latest formulation has been known for its capability to avoid volumetric locking in fully incompressible and nearly incompressible materials. It has been first developed by [9] for dimensionally reduced element with three-dimensional stress state, i.e. plain strain and axissymmetry elements, but not for the linear tetrahedral elements. The formulation states that shear deformation is considered as independent variables from the influence of hydrostatics or volumetric pressure. Therefore, in case of high bulk modulus κ obtained shape changing due to shear will not be locked by such pressure.

This article consists of five sections including this section. Section 2 describes theoretical basis of strain energy used in this work in order to characterize energy convergence criteria with respect to mesh size. Bringing the criteria to the linear element will result in the VMR. Next, Section 3 discuss on implementation of algorithm to tackle auto VMR method. Furthermore, the algorithm is validated in Section 4 using classical benchmarking problems namely cantilever beam and shell torsion with discussion on the results. Finally, Section 5 outlines reviews on the VMR based on the results from Section 4.

Mesh Refinement on Tetrahedron Volume

To start with, an in-house nonlinear finite element solver has been developed using FORTRAN and the G95 Compiler [10] with additional interfaces for pre- and post-processing in the Gmsh [11]. The solver solves simultaneously residual equations $\mathbf{r}_u$ and $\mathbf{r}_p$ of the mixed u/p formulation [9] over tetrahedral element volume V under the Cartesian coordinate system $\mathbf{x}$ denoted by

$$\mathbf{r}_\mathbf{u} = \int_V \frac{\partial \boldsymbol{v}}{\partial \mathbf{x}}^T \mathbf{F}\left[\mathbf{T} - \mathrm{pJ}\mathbf{C}^{-1}\right]\mathrm{dV} - \mathbf{f}_{\mathrm{ext}} \tag{1}$$

$$\mathbf{r}_\mathrm{p} = -\int_V \left[(\mathrm{J}-1) + \frac{\mathrm{p}}{\kappa}\right]\mathrm{dV} \tag{2}$$

where $\mathbf{v}$, $\mathbf{F}$, $\mathbf{T}$ and f_{ext} are defined as 9x1 virtual deformation vector, 9×6 deformation gradient matrix, 6×1 deviatoric 2nd-Piola Kirchoff stress in the Voigt notation and external forces, respectively. While the unknown 9×1 vector of the inverse right-Cauchy deformation $\mathbf{C}^{-1}$ is evaluated at tetrahedral nodes, additional pressure variable p is computed for one extra node at the tetrahedral centroid. Material compressibility J is measured using $\mathrm{J} = \sqrt{\det \mathbf{C}}$ where special case of J = 1 means fully incompressible behavior imposed during material deformation. In order to find solutions of nonlinear Equations (1) and (2), the full Newton-Raphson iterative scheme with load and deformation control is implemented in the solver.

Meanwhile, internal or strain energy W over the volume of tetrahedral is given as

$$W(\mathbf{E}) = \frac{1}{2}\int_V \mathbf{E}^T \mathbf{C}_T \mathbf{E} dV \tag{3}$$

where $\mathbf{E}$ and $\mathbf{C}_T$ are the Green-Lagrange strain and tangent stiffness of the Mooney-Rivlin hyperelastic model, respectively. In case of the mixed u/p formulation, additional pressure variable p can be considered in Eq. (3) as

$$W(\mathbf{E}) = \frac{1}{2}\int_V \mathbf{E}^T \left(\mathbf{C}_T \mathbf{E} - pJ\mathbf{C}^{-1}\right) dV. \tag{4}$$

Tetrahedron volume used here is a part of six tetrahedral which build a hexahedral volume with unit length as shown in Figure 1. Since development of this method is still ongoing, this work is merely intended to discuss underlying fundamental idea and to leave the influence of arbitrary tetrahedral configuration for future research. The volume in Figure 1 is then discretized with tetrahedral elements for different number of elements, i.e., 1, 9, 28, and 533. Single force is applied to node 4 (see Figure 1 for its exact location) in specified direction with the remaining nodes fixed.

The force direction is made intuitively for both parallel and perpendicular with respect to triangular plane formed by the constrained face. With N_i and i denoting number of element used and its index in different refinement step, respectively, a new parameter r_{Vi} is defined with respect to the 1 element mesh to form relationship $r_{Vi} = N_i/N_1$.

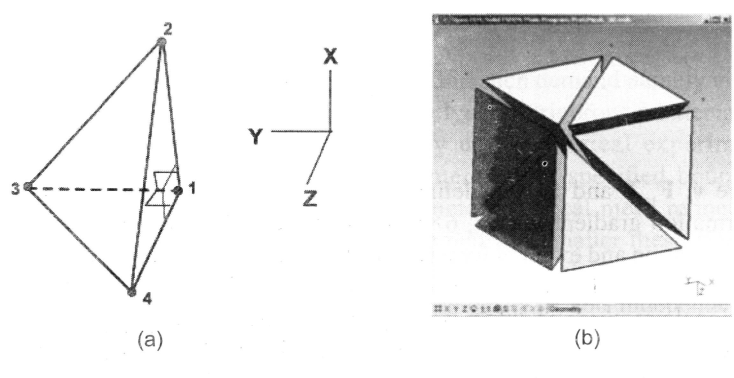


Figure 1: Tetrahedron volume (a) used as the meshed refinement domain is a part of a cube (b) consist of six tetrahedral volumes

Deformation shape of the tetrahedron volume after simulation can be seen in Figure 2. Here, the corresponding strain energy W_i on each refinement step i are normalized with respect to the energy of 1 element, i.e., $r_{Wi} = W_i/W_1$, to obtain a power law relationship as shown in Figure 3.

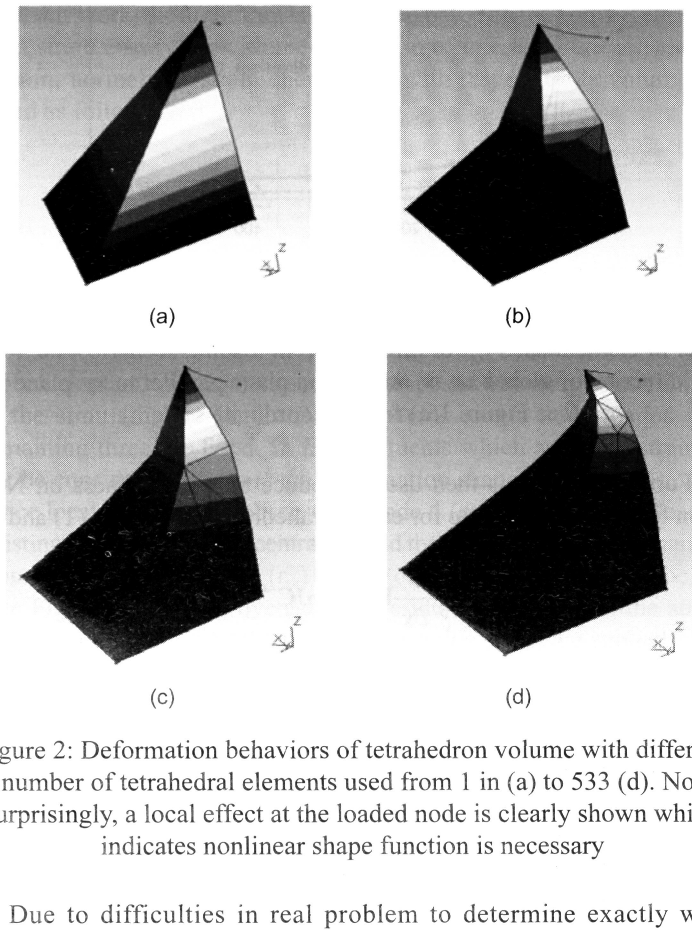


Figure 2: Deformation behaviors of tetrahedron volume with different number of tetrahedral elements used from 1 in (a) to 533 (d). Not surprisingly, a local effect at the loaded node is clearly shown which indicates nonlinear shape function is necessary

Due to difficulties in real problem to determine exactly whether tetrahedral elements in consideration tends to behave as the above loading scenario or not, the best choice for approximation in Figure 3 will be optimized simply by considering the power equation

$$r_{Wi} = a\, r_{Vi}^{b} \tag{5}$$

where a and b can be determined by finite element convergence criteria. This issue will be discussed in the following section.

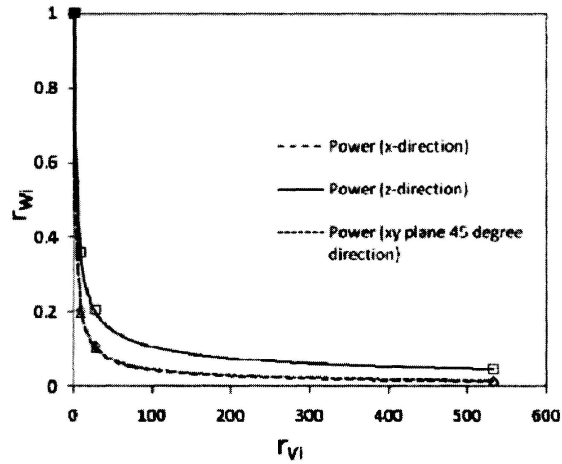


Figure 3: Power law relationship between strain energy ratio r_{Wi} and ratio of number of element used r_{Vi} for three different loading scenarios, i.e. force in direction of global x-, z-, and 45° on plane parallel to xy-plane (see Figure 1(a) for the coordinate system)

Furthermore, r_{Wi} is then used to reduce tangent stiffness on Newton-Raphson linearization scheme for each tetrahedral expressed in (1) and (2) as

$$\mathbf{r}_u = r_{Wi} \int_V \frac{\partial \boldsymbol{v}^T}{\partial \mathbf{x}} \mathbf{F}\left[\mathbf{T} - pJ\mathbf{C}^{-1}\right] dV - \mathbf{f}_{ext} \quad (6)$$

$$r_p = -r_{Wi} \int_V \left[(J-1) + \frac{p}{\kappa}\right] dV. \quad (7)$$

Both equations explain that using the equal external energy and requesting $r_{Wi} < 1$ larger deformation field u must be exist to maintain the corresponding internal energy equal to the external one. Hence, they indicate also a refinement process without additional degree of freedom or simply called “virtual mesh refinement” compared to what observed in common mesh refinement process, i.e., by adding “physical” element to arbitrary domain.

Auto Virtual Mesh Refinement (VMR)

Through this section an algorithm to tackle automatic VMR based on result in the Section 2 will be presented. In the sense of adaptive analysis or refinement tasks in FEM, an advantage of implementing VMR is apparent because working on re-meshing domain of interest is not necessary anymore. The advantage can be achieved by developing special algorithm which can detect convergence behavior from criteria in demand, e.g., global strain energy or local stress concentration, in order to avoid deformation field running into overestimating results. This issue is important because finite element solutions are always necessary to be bounded by their respective analytical or continuous solutions.

In this work, the algorithm is developed based on the convergence criteria of global strain energy per-volume in domain of interest. For each element in the domain, normalized strain energy W^*_{int} with respect to its volume can be computed as follow

$$W^*_{int}(\mathbf{E}) = \sum_{n=1}^{k}\left[\frac{1}{2V}\int_V \mathbf{E}^T\left(\mathbf{C}_T\mathbf{E} - pJ\mathbf{C}^{-1}\right)dV\right]_n \tag{8}$$

where n and k denote element numbering and total number of element, respectively. In order to manage the algorithm obvious and simple three basic assumptions below must be noted, i.e.:

a. Not all elements in a domain sense the imposed boundary condition similar to the simulation in Figure 2, i.e., having single node loaded and the remaining three are fixed. In fact, elements which are not restrained can not be considered to mimic the deformation shape in Figure 2.
b. Since local deformation observed at node 4 (see Figure 1) may lead to the existing of local strain concentration and the remaining larger domain is free from it, different function $r_{Wi}(r_{Vi})$ to drive convergence process must be defined (see Figure 4) to avoid overestimate results with respect to the analytical solution. In order to find the function these rule below are applied

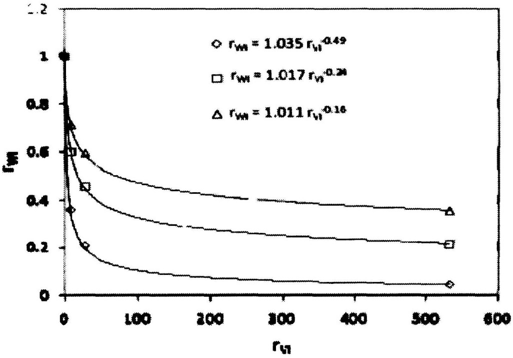


Figure 4: Three different values r_{Wi} as functions of r_{Vi} which represent different possible reduction of r_{Wi} due to different actual condition of each element in domain

i. $r_{Wi} = 1.011 r_{Vi}^{-0.16}$ is related to $r_{Wi} = (W_i/W_1)^{1/3}$
ii. $r_{Wi} = 1.017 r_{Vi}^{-0.24}$ is related to $r_{Wi} = (W_i/W_1)^{1/2}$
iii. $r_{Wi} = 1.035 r_{Vi}^{-0.49}$ is related to $r_{Wi} = (W_i/W_1)$.

The power of 1/3, 1/2 and 1 in r_{Wi} are chosen merely based on regularity and not derived by rigorous mathematical analysis. While the rule (iii) is used for the elements in the area which sense high stress concentration, the rule (ii) and (i) are dedicated for other location in the domain with relatively close and far away from the area, respectively.

With $r_{Wi}(r_{Vi})$ in hand, (6) and (7) can be solved using Newton-Raphson scheme.

Finally, the algorithm implemented in this work can be seen in Figure 5 where the convergence criteria is based on W^*_{int} by considering equation

$$\Delta W^*_{int} = \frac{W^*_{int}\left(r_W^{i+1}\right) - W^*_{int}\left(r_W^{i}\right)}{W^*_{int}\left(r_W^{i}\right)} \leq 0.05 \tag{9}$$

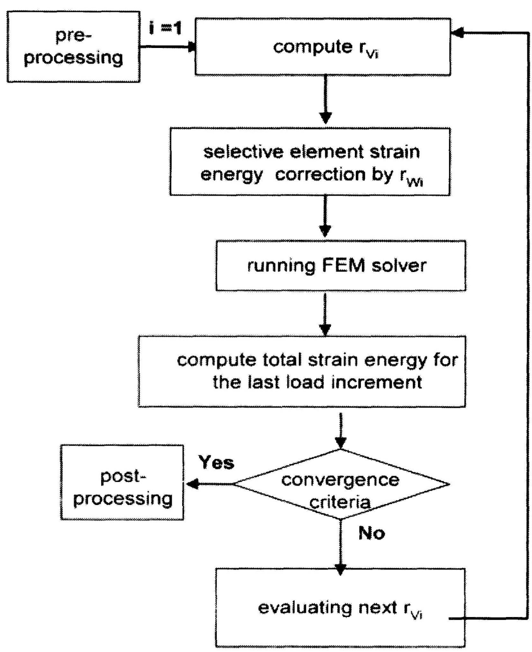
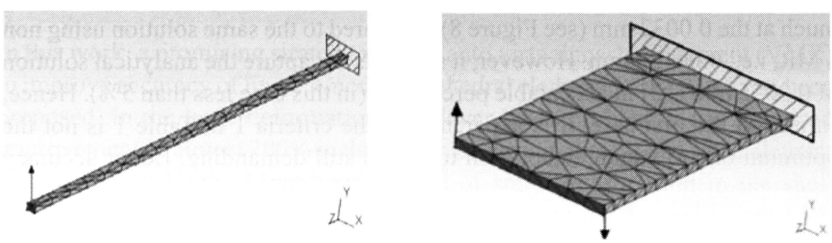


Figure 5: Algorithm of the auto VMR

Numerical Experiments: Cantilever and Shell Structures

In order to study accuracy of the proposed method, two classical benchmarking cases with their existing analytical solutions, i.e., large deformation in cantilever beam [12] and shell with torsional load [13], are introduced here as shown in Figure 6. It is widely known that using linear 4-node tetrahedral for such cases is not recommended due to over-stiff responses. While the first case is mainly dedicated to show capability of the proposed method in capturing large deformation shape, the second one is aimed to show the capability in small deformation for thin shell structures.

At the first case, the cantilever beam with rectangular cross section 0.15 m × 0.1m and length of 10 m is loaded by 281.4 N forces at its tip perpendicular to the length. Since large deformation is the main consideration here, the Kirchoff-Venant hyperelastic material model is used with modulus elasticity E = 100 MPa and the Poisson ratio $v = 0$. Two different bounds on the internal strain energy are implemented herein (see Table 1). Results from the simulation and benchmarking process are shown in Figure 7 where the existing analytical solution for the same case by [12] is used as the reference solution.



(a) (b)

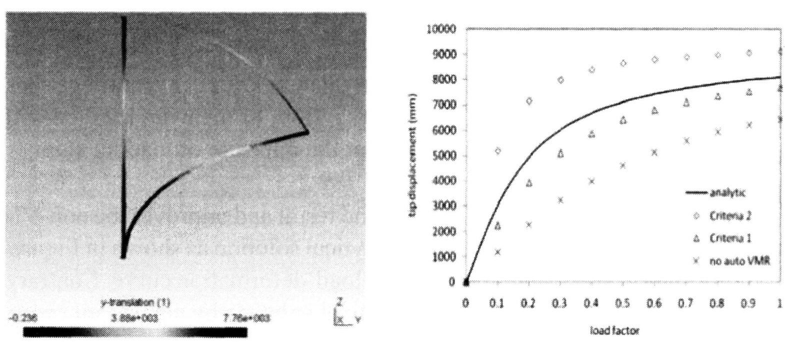
Figure 6: Two benchmarking cases used in this work, i.e., (a) cantilever beam and (b) thin shell with torsional loading (b). Both of them are meshed with 4-node tetrahedral element and fixed at the opposite of loading sides

As predicted, the criteria 1 yields good result and improves the non-VMR solution very much with respect to the analytical solution as shown in Figure 7a for its deformation shape and Figure 7b for load-deformation curve. Contrary to this, the criteria 2 overestimates the analytical solution by significant order of magnitude and leaves problem with analytical bounded requirement. Moreover, the area between both curves indicates possibility to find optimum criteria to find better approximation to the analytical solution.

Table 1: Convergence stop criteria

r_{Wi}	Criteria 1	Criteria 2 (only for cantilever)
$= 1.011\ r_{Vi}^{-0.16}$	$0 < \dfrac{W^*_{int}}{(W^*_{int})_{max}} < 0.5$	$0 < \dfrac{W^*_{int}}{(W^*_{int})_{max}} < 0.01$
$= 1.017\ r_{Vi}^{-0.24}$	$0.5 \leq \dfrac{W^*_{int}}{(W^*_{int})_{max}} < 0.8$	$0.01 \leq \dfrac{W^*_{int}}{(W^*_{int})_{max}} < 0.1$
$= 1.035\ r_{Vi}^{-0.49}$	$0.8 \leq \dfrac{W^*_{int}}{(W^*_{int})_{max}} < 1$	$0.1 < \dfrac{W^*_{int}}{(W^*_{int})_{max}} < 1$

In the second case, a thin shell with length of 2 mm, width of 1 mm and 0.05 mm thickness is loaded with 1 N force acting coupled to form pure torsion. Material used here is the Kirchoff-Venant with E = 5851851 MPa and $\nu = 0.46$. Only the criteria 1 is used here to obtain convergence result. In this small deformation case, the auto VMR has improved the tip deformations very much at the 0.0032 mm (see Figure 8) compared to the same solution using non VMR, i.e., 0.00129 mm. However, it still fails to capture the analytical solution of 0.0055 mm [13] in acceptable percentage (in this case less than 5%). Hence, similar to the first case, it is apparent that the criteria 1 in Table 1 is not the optimum one to which elaboration to this is still demanding. Better accuracy



(a) (b)

Figure 7: Deformation shape of the beam after simulation using the criteria 1 (a) with the corresponding deformation-load curves for the criteria 1 and 2, analytical solution [12], and non VMR (b)

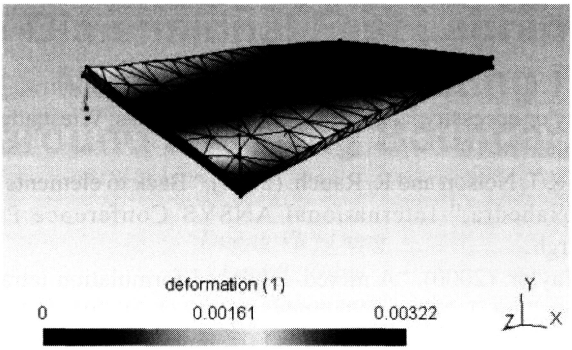


Figure 8: Result for the shell torsional simulation with its deformation from the auto VMR

can be done by applying $r_{Wi} = 1.035\ r_{Vi}^{-0.49}$ for more elements by changing the interval for the $W^*_{int}/(W^*_{int})_{max}$, for instance.

Concluding Remarks

In this work, a promising strategy namely auto virtual mesh refinement (VMR) to improve accuracy of linear 4-nodes tetrahedral element used in CAE has been proposed. In the large deformation cantilever beam problem it has produced improvement of almost 200% in the deformation compared to the same element with non-auto VMR. Moreover, in case of small deformation in the shell structure stipulated by torsional coupled forces, it has also improved deformation significantly to almost 200% from the non-auto VMR solution. Hence, from both classical problems where the 4-node linear tetrahedral element is known to have very stiff responses indicate further potential implementation of the underlying idea of the auto VMR method. Finally, according to the benchmarking with the analytical solutions optimum criteria to control convergence behavior of the auto VMR is necessary to be determined. This can be a challenging topic for further research objectives.

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